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Three-Dimensional Wakes of Simulated Wind Turbines

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An experimental and theoretical investigation of the wake behind a porous disk which simulates the effect of a wind turbine is presented. Modeling of the wake in an environmental wind tunnel is described and experimental results for the three-dimensional model wake flowfield are presented. A three-dimensional turbulent flow analysis is developed to deal with prediction of the wake characteristics. The theoretical results are shown to display the experimentally observed features of the wake.

I. Introduction

THERE is increasing need for accurate prediction of the interaction of wind turbine generators (WTG's) in arrays, as evidenced by literature published both here[1,2] and abroad.[3] The problem is the determination of the nature of the wake of a WTG and its development downstream prior to possible interaction with another WTG in the array. This is a case of relaxation of a wake in a turbulent shear flow, which is an important basic problem in fluid mechanics and, as such, deserves attention and study. Field tests are being carried out both here and abroad, but such approaches are expensive, difficult to carry out, and by their nature are incapable of being completely controlled. The lack of control contributes to the difficulty of interpreting such experiments and to the hazards of extrapolating from such results.

To alleviate this problem it seems important to carry out a coupled experimental and theoretical program on laboratory-scale model wakes for the following reasons: First, such a laboratory investigation would provide information concerning model turbine wake characteristics over a wide range of controlled upstream conditions. Second, wake information thus obtained would be of value for checking the observations of actual wake characteristics and the predictions of various models to be developed in and for projected large-scale programs. Third, a relatively inexpensive tool for continuing assessment of wind turbine flowfields would be in hand.

An exploratory investigation of laboratory-scale simulated WTG wakes has been carried out by Sforza et al.[4] Results of that study are included here for completeness. Other studies have concentrated primarily on the overall effects of clusters of WTG's on global performance rather than on the detailed fluid dynamics of the wake re-energization process, i.e., Refs. 5-25. The objective of the present investigation is to establish the fluid dynamic processes occurring in the development of the flow in the wake, to determine the importance of some of the various simulated atmospheric boundary-layer characteristics on the flow, and to develop a reasonably accurate prediction method for the calculation of the simulated wake flowfield.

II. WTG Wake Flowfield

The wake behind a two-dimensional or axisymmetric body is a classical fluid dynamic problem and is discussed in many textbooks (e.g., Schlichting[26]). However, the accurate prediction of the characteristics of even these very simple flows is difficult to ensure, as the results of the NASA Free Turbulent Shear Flow Conference[27] show. Therefore, it is important to recognize that the much more complicated wake characteristics of a WTG will be quite difficult to predict as well as to measure. A schematic diagram illustrating some of the various effects to be encountered in WTG wakes is presented in Fig. 1. These effects will be examined sequentially and related to some relevant investigations by previous researchers in order to provide a foundation for studying the complete problem.

Thrust Effects

The extraction of power from the flow involves the application of a force on the rotor by the fluid. In the terminology of fluid dynamics, this is a drag force since it acts in the direction of the undisturbed freestream velocity. In wind energy terminology, this force is called the "thrust." Since this thrust is represented by the deficit in momentum flux in the wake caused by the presence of the rotor, it is the major factor which determines the nature of the wake flow. A simple quasi-one-dimensional momentum analysis which illustrates these comments is given by Wilson et al.[28] The thrust (drag) effects on the wakes of disklike bodies, axisymmetric or not, are fairly well documented, e.g., the experimental studies of Cooper and Lutsky[29] and Kuo and Baldwin.[30] Fluid dynamic analytic fundamentals of wake flows are presented by Chang,[31] who shows the various effects on the wake due to forces exerted on the fluid by the body in question. Turbulence modeling and calculation techniques for different free-shear flows, including wakes, are presented in Ref. 27.

Three-Dimensional Mixing Effects

Due to the presence of a drag-producing tower, among other things, the wake produced by a WTG will not be axisymmetric. Therefore, considerations of three-dimensional (3-D) turbulent mixing effects will have to be made. Analytic study of such 3-D effects have been performed by Sforza et al.,[32] Sforza,[33] Steiger and Bloom,[34,35] and Kuo and Baldwin.[30] One of the major consequences of three-dimensional turbulent mixing in both jets and wakes is that the velocity deficit in, say, a 3-D wake recovers in the fashion of an axisymmetric wake, but the growth of the wake boundaries in the two geometries is very different. This has been discussed by Trentacoste and Sforza.[36]

Wake Rotation Effects

The flowfield induced by the rotor blades includes a swirling motion imparted to the wake. Swirling wakes have been studied by Steiger and Bloom[37] and swirling jets by Chigier and Chervinsky.[38] Such swirling flows are similar to aircraft trailing vortices which have been extensively investigated, e.g., Ref. 39. From these studies one may conclude, as a first approximation, that the effects of swirl will be

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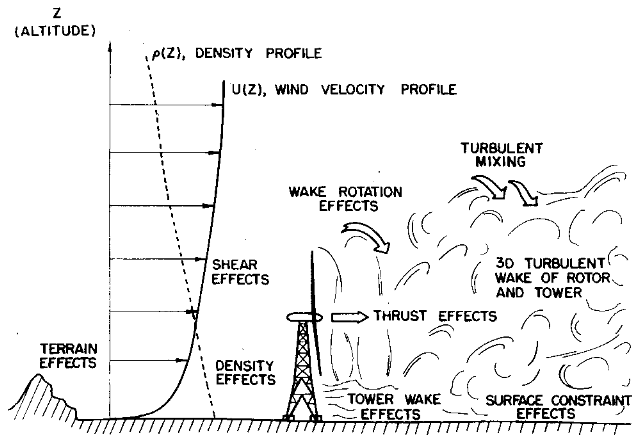

Fig. 1 Schematic diagram of WTG wake flow and the various effects to be expected in practical situations.

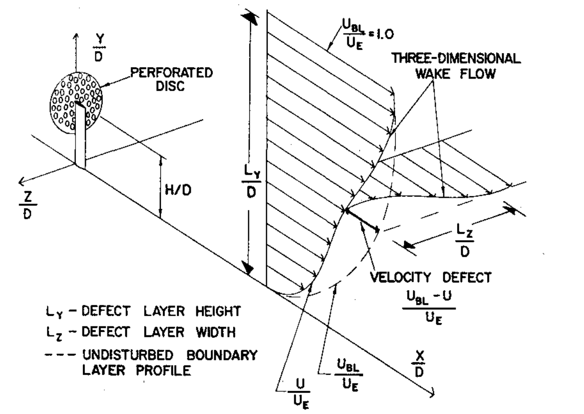

Fig. 2 Coordinate system used in the present study. Also shown are the various definitions used in the text.

of secondary importance in terms of wake re-energization. This is due to the fact that swirl persistence is, in general, weak and decays swiftly, unless the swirl momentum is on the order of the mainstream momentum in the wake. On the other hand, localized swirling flows such as the tip vortices from the rotor blades can be quite persistent, especially under conditions of low atmospheric turbulence. The interaction of these vortices with downwind WTG's or other structures can be a problem of concern since the localized vortices can be a periodic concentrated forcing function which may induce vibration and fatigue in such cases. Since this interference is not directly related to wake re-energization, it will not be addressed here but rather deferred to other programs of research.

Surface Constraint Effects

The proximity of the rotor to the ground ensures that the rotor wake will interact therewith, causing additional complications. This type of interference has been studied for the case of a solid three-dimensional obstacle mounted on a ground plane by Sforza and Mons.[40] It was found in that experimental and analytic investigation that rather strong interactions influenced the re-energization of the local boundary-layer flow. Other studies of a similar type have been presented by Schubauer and Spangenberg[41] and by Sedney.[42] Generally, the effect of surface constraint is to alter the mixing properties of the wake by blocking mass entrainment through the underside of the wake. In addition, the three-dimensional character of the wake cross section is aggravated and the re-energization process is altered by the presence of the ground.

Shear Effects

The presence of a vertical gradient of velocity in the atmospheric boundary layer can affect WTG operation. It will change the mixing properties of the wake of the WTG as well. Wakes in shear flows have not been the subject of much attention except in atmospheric pollution problems. An early experimental work of basic interest is that by Eskinazi.[43] Frost et al.[44] have developed a boundary-layer approach to treat the shear flow over obstacles in the natural wind. Other studies pertaining to effects induced by the atmospheric boundary-layer velocity gradient are given in Refs. 45-48.

Ambient Turbulence Effects

The atmospheric boundary layer has a turbulence structure that depends upon many factors.[45,46] This flow interacts with the rotating blades of a WTG to form a wake in which the turbulence structure is further complicated. Of major importance in the wake re-energization process is the effect of this external turbulence of the surrounding atmospheric boundary-layer flow on the mixing and entrainment rate in the wake. It is expected that high turbulence levels will tend to speed up the wake recovery, but it is not clear what the magnitude of the change will be. This facet of the WTG wake problem is of high priority, because the development of a turbulence model for accurate wake flow predictions will depend strongly on the nature of the external flow turbulence and on the turbulence structure of the wake itself.

Density Effects

The atmospheric boundary layer is also characterized by density gradients normal to the surface. These gradients can become important in determining the nature of fluid motions, as described by Turner[49] and Long.[50] There has been some interest in wakes in density-stratified media because of the application to submarine operations. It has been found that substantial changes in wake growth, the so-called "wake collapse," can be brought about by density gradients in the undisturbed fluid.[51,52] Such buoyancy effects might alter the mixing and re-energization process in WTG wakes.

Terrain Effects

The nature of the surface in the intervening space between elements in a WTG array also influences the WTG wake characteristics. Terrain effects on the approach flow to a WTG have been reviewed by Meroney et al.[53]; however, the character of the interactions between the wake of a WTG and terrain features has not been addressed. Of course, when the surface aft of a WTG is essentially flat, the effect is that of surface constraint, as discussed previously. Undulations in terrain, or the presence of structures, may have an important effect on wake developments.

We have described some of the more important aspects of the many different factors which may influence the wake of a large WTG. It becomes clear that it would be worthwhile to be able to exercise some degree of control over these various factors in order to differentiate the magnitude of each. Such control would yield the flexibility necessary to formulate a fairly general predictive capability embracing the many different applications and sites of WTG arrays.

III. Experimental Approach

The experimental facility utilized in the present study is the Environmental Wind Tunnel of the Polytechnic's Aerodynamics Laboratories. It has a test section measuring 1.2×1.5 m in cross section and 6.2 m in length and operates in the 0.1 to 10 m/s range. This tunnel has the capability of producing vertical velocity and density gradients appropriate to atmospheric boundary-layer simulation, as well as uniform velocity and density profiles for conventional wind tunnel investigations.

Measurements of the mean velocity field in planes normal to the flow direction are made by means of pitot-static tubes and variable reluctance transducers, while turbulence measurements are made with a hot-wire anemometer. Data presented here were taken in planes normal to the flow direction, at streamwise stations of $X/D=2$, 5, 7.5, 10, 12.5, and 15, at a constant nominal tunnel velocity of 4.3 m/s. At each test station the velocity was measured at the node points of a 25×25 mm grid in the Y and Z directions. For all cases the experiments were performed at a hub (i.e., disk center) height to diameter ratio (H/D) of 1.0. The maximum velocity defect and wake growth in the Y and Z directions as a function of streamwise distance was determined from the extensive velocity data for the three cases considered. A schematic diagram of the porous disk mounted on a streamwise strut, as used in the experiment, is shown in Fig. 2 along with descriptions of the wake parameters.

Experimental Model

The object of the investigation is to study the wake of a suitable laboratory model of a WTG incorporating as many of the various conditions likely to be encountered in practice as possible. Since the thrust of the WTG is the major factor in wake development, it is this force which must be adequately modeled. From a simple one-dimensional actuator disk analysis, such as that presented in Ref. 28, we find that the thrust

$$T=4a(1-a)\,\tfrac{1}{2}\rho A U_\infty^2$$

We may define a thrust coefficient

$$C_T \equiv \frac{T}{\tfrac{1}{2}\rho A U_\infty^2}=4a(1-a)$$

where a is the axial interference factor, which, in turn, is defined as

$$a \equiv (U_\infty - U)/U_\infty$$

U_∞ is the undisturbed uniform wind speed, and U is the flow speed through the actuator disk.

The question remaining at this point is: How do we model this force? The actual WTG is in a highly turbulent field, thus its wake is completely turbulent also, so that Reynolds number matching is unnecessary to first order. The neglect of viscous effects in such cases is discussed by Townsend.[54] However, the modeling oO the rotor itself is not readily achieved. The development of a small model rotor that will reproduce the appropriate range of power and thrust coefficients at the correct tip speed ratio is not achievable without altering other aspects of the flowfield. For example, matching the high tip speed ratios of a large WTG requires rotational speeds of a small-scale model rotor in the neighborhood of 500 s^{-1}. This can be shown to introduce significant radial pressure gradients in the model wake, which adversely influences the development of an accurately scaled wake. Furthermore, since wake rotation is not considered to be one of the more important features of the re-energization of a WTG wake, it is felt that the thrust may be produced by an appropriate stationary model that has the correct geometric shape, permits flow-through, and allows an appropriate C_T variation. Such model characteristics can be provided by a porous disk constructed of a perforated material or a mesh.

The model disk diameter for our wind tunnel was chosen to be 15 cm so that wake traverses up to 15 disk diameters downstream are possible. Disks of different porosities are used to simulate a range of rotor thrust loadings. The disk is mounted on a streamlined strut, although a modeled tower could also be used. It is estimated that the blockage area of the models is around 2% or less of the cross-sectional area of the wind tunnel, which should ensure minimum blockage effects, according to Merony and Yang.[55] Hence, the disk is small enough to allow very small wind-tunnel blockage effects, but large enough to assure adequate spatial resolution of the wake flowfield. Such a model has been found to provide a mean wake defect signature which is of a character similar to that found in the few existing data for large-scale machines, e.g., Ref. 15.

Of course, such a model does not accurately simulate all detailed effects, including those of wake spatial periodicity, imbedded tip vortices, and imbedded blade-generated turbulence. On the other hand, it does retain analogs of some of those effects on a global basis. It is expected that a model of this type is certainly applicable to the type of fundamental approach to this complicated fluid dynamic problem, and can reveal important features of the flow pertinent to practical situations.

Atmospheric Simulation

The environmental wind tunnel described previously was operated in the constant density, vertical shear mode. The atmospheric boundary layer over flat open country can thus be simulated with respect to velocity gradient, although the temperature is restricted to a constant value and thus relates to neutral stability conditions. Two vertical velocity profiles have been studied and are shown in Fig. 3 in terms of U/U_E as a function of Y/Y_E, where the subscript E denotes conditions at the edge of the boundary layer. A reasonable fit of a power law profile to the data shown in Fig. 3, i.e., $U/U_E=(Y/Y_E)^p$, yields $p=0.16$ and $p \simeq 0$. The former is the value given by Plate[45] as indicative of wind profiles over flat open country, while the latter is a value indicative of essentially no wind shear. This last case has been studied because it is one readily achieved in most wind tunnels and it will yield some idea of the effects vertical shear has on the wake re-energization process.

In Fig. 3 the values of Y_E are 0.5 and 300 m for the wind tunnel and atmospheric flow, respectively. In the experiment we use a 15 cm-diam perforated disk to represent the rotor, which in the large-scale case corresponds to a rotor with a sweep diameter of about 100 m. Thus, we expect the results of this study to have relevance to WTG arrays of utility size.

Initial Profile

The model used to simulate the rotor under conditions of high thrust (simulating high-power loading) is a masonite disk 6 mm thick by 152 mm in diameter in which many holes, 6-10 mm in diameter, are drilled to provide for flow-through. The porosity of the disk (open area to disk area ratio) is 0.43, and the thrust coefficient of the porous disk is a moderate 0.5 for the case where a vertical velocity gradient exists and a high 0.92 for the case of a negligible velocity gradient. The model used to simulate the rotor under low thrust conditions is a

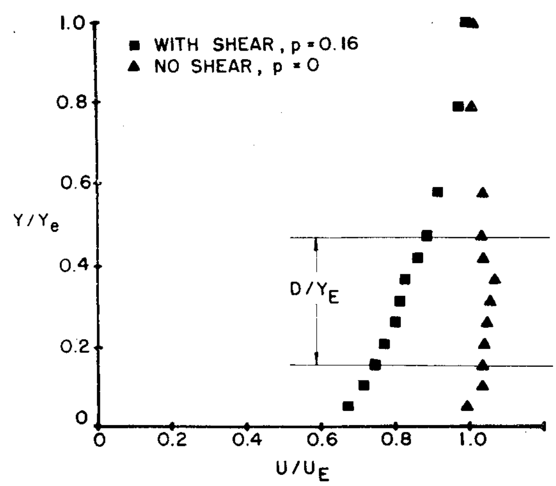

Fig. 3 Undisturbed flow cases studied in the present program showing nondimensional streamwise velocity as a function of distance normal to ground. Location of disk of diameter D in experiments is shown.

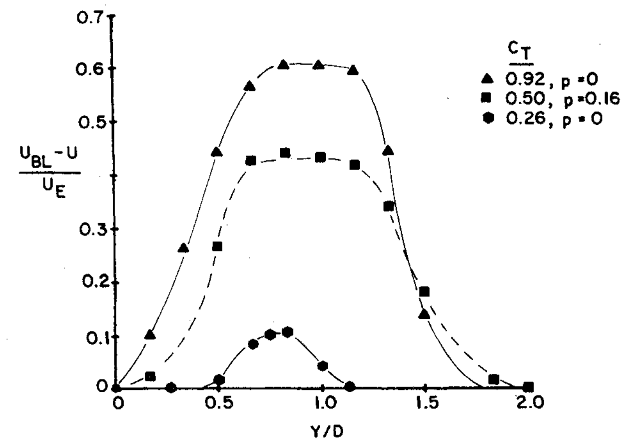

Fig. 4 Velocity defect in the vertical plane passing through the center of the simulated rotor for the three cases of high, moderate, and low thrust loading at the streamwise location $X/D=2.0$. Lines faired through data for clarity.

disk of wire mesh 152 mm in diameter with a 12.7 mm square mesh, a porosity of 0.83, and a thrust coefficient of 0.26.

The loading or thrust on the model was calculated by integration of the momentum deficit between the wake profile, in the plane $X/D=2.0$, and the undisturbed upstream profile. The thrust coefficient was taken as the ratio of the wake thrust to the product of freestream dynamic pressure at the hub height ($H/D=1$) and the cross-sectional area ($\pi D^2/4$) of the model. The disk is mounted on a streamlined strut of symmetrical airfoil cross section, since, for exploratory tests, we did not wish to introduce the complication of a truss tower. Experiments were performed for a hub H/D of 1.0, although tests at different ratios could easily be performed.

Profiles of the velocity defect, $(U_{BL}-U)/U_E$, at the initial station, $X/D=2.0$ for the three cases studied here, i.e., high loading with no shear, moderate loading with shear, and low loading with no shear are shown in Fig. 4. These profiles represent the difference between the undisturbed boundary-layer profile and the actual velocity distribution measured behind the disk at that station, in the centerline plane, $Z=0$. The location $X/D=2.0$ was chosen as the plane of the initial profiles for both the theory and experiment because it was close enough to the model to permit an extensive downstream range of investigation while far enough from the model so that flow details particular to the porous disk are negligible. From the latter it may also be inferred that $X/D=2.0$ lies beyond the region wherein recirculation and/or pressure gradient effects are important.

IV. Analysis

There are many levels to the analysis of turbulent shear flows such as the simulated WTG wake considered here. It is pertinent to consider the remarks of the NASA Free Turbulent Shear Flow Conference[27] Evaluation Committee: "... the complexity required in turbulence modeling depends upon the problem. That is, the selection of a (calculation) method should depend upon a careful determination of the information required to resolve the particular needs of the user."

In the WTG wake problem, interest is focused on the growth of the wake boundaries and the re-energization of the wake, i.e., the decay of the defect velocities in the wake. Though the various practical constraints such as wind shear are essential to the problem and can be readily modeled, the detailed nature of the turbulent structure of the wake is more difficult to treat. Therefore, it seems appropriate here to utilize a simple turbulent exchange model, while retaining, to as great an extent as possible, a rigorous description of the other characteristics of the flow such as three dimensionality.

Basic Model

The flowfield we consider is shown schematically in Fig. 2. A two-dimensional turbulent shear flow approaches a flat perforated disk; this obstacle, standing normal to and located at some height above the ground plane, disturbs the oncoming stream. The flowfield behind the obstacle will have a wakelike character which we wish to exploit.

Sforza and Mons[40] treated a three-dimensional wake developing within a two-dimensional turbulent boundary layer by means of a linearized analysis which led to good agreement between theory and experiment. Their linearized approach was based upon earlier developments discussed by Hinze[56] and Clauser[57] and is described in some detail in Ref. 40. In essence, the boundary-layer flow and the wake immersed within it are both treated as small perturbations to a uniform freestream flow. Then, since the equation describing the flow under these assumptions is linear, the solution to the combined problem of the wake within the boundary layer is found by superposition.

In the present case then, we need only consider the three-dimensional wake perturbation and its development downstream. This wake perturbation, as calculated from the linearized equation, will represent the difference between the actual shear layer profile in the downstream region and the undisturbed shear layer profile. This can best be visualized by referring to Fig. 2 where the wake defect is clearly indicated.

The analysis uses an eddy viscosity model to characterize the turbulent transfer process. The advantages and disadvantages of the eddy viscosity approximation are well known. It is only necessary to indicate that such an approximation is convenient and has been utilized with success for predicting the velocity field of wake flows. The eddy viscosity can be modeled with varying degrees of complexity and accuracy as shown by Launder and Spalding.[58] Finally, the effects of pressure gradient will be neglected in the analytic model. The presence of obstacles does induce pressure gradients which are generally on the order of the experimental error, at least in the regions of interest.

Application to the Wake

The approach outlined above is used for the simulated WTG wake problem. In this case, the lowest-order equation for the streamwise momentum flux follows from a linearization of the flowfield equations and the assumption of an eddy viscosity model for turbulent transfer:

$$U_E u_x = (\epsilon_l u_y)_y + (\epsilon_2 u_z)_z \qquad (1)$$

Here the local mean velocity $U=U_E-u$, U_E is the velocity at the edge of the boundary layer, $u \ll U_E$, and ϵ_l and ϵ_2 are the turbulent viscosities in the transverse directions. This is similar to the classic basic wake flow development (see Schlichting[26]), but for three-dimensional mixing. The important factor here is the deduction of semiempirical or theoretical relationships for ϵ_l and ϵ_2 that will provide an accurate flowfield description.

We introduce normalized variables

$$\xi = X/D, \qquad \eta = Y/D, \qquad \zeta = Z/D \qquad (2)$$

where D is the diameter of the disk obstacle. It is assumed that the coefficients of the diffusive terms in Eq. (1) are equal and functions of X alone, i.e.,

$$\epsilon_l(X) = \epsilon_2(X) = \kappa u_m \sqrt{L_Y L_Z} \qquad (3)$$

where κ is an empirical constant determined from experiment, u_m is the maximum velocity defect and, therefore, a function of X, and L_Y and L_Z are the widths of the wake as shown in Fig. 2, and are also functions of X. Here L_Y is measured in the plane $Z=0$, while L_Z is measured at the Y station where $u=u_m$. The eddy viscosity formulation in Eq. (3) is of a form

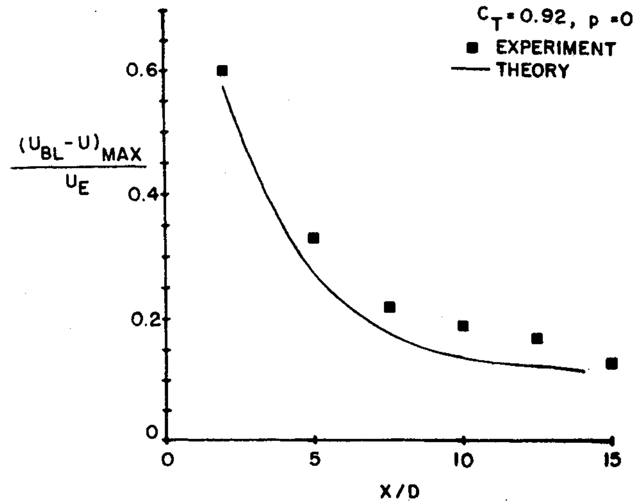

Fig. 5 Maximum velocity defect as a function of streamwise distance for the case of high thrust loading and no upstream shear.

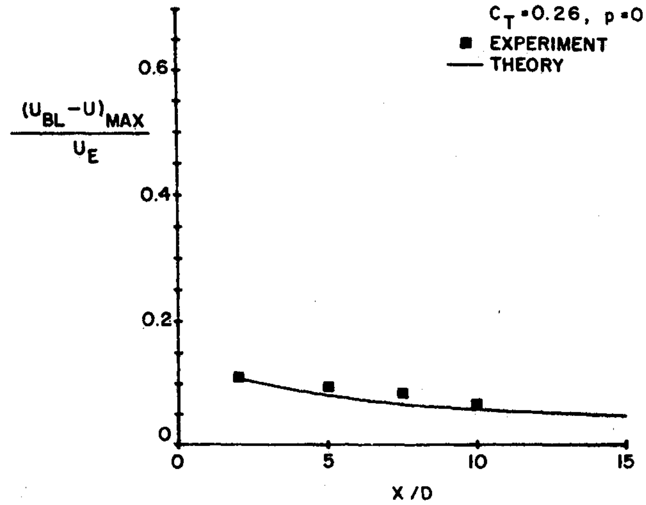

Fig. 7 Maximum velocity defect as a function of streamwise distance for the case of low thrust loading and no upstream shear.

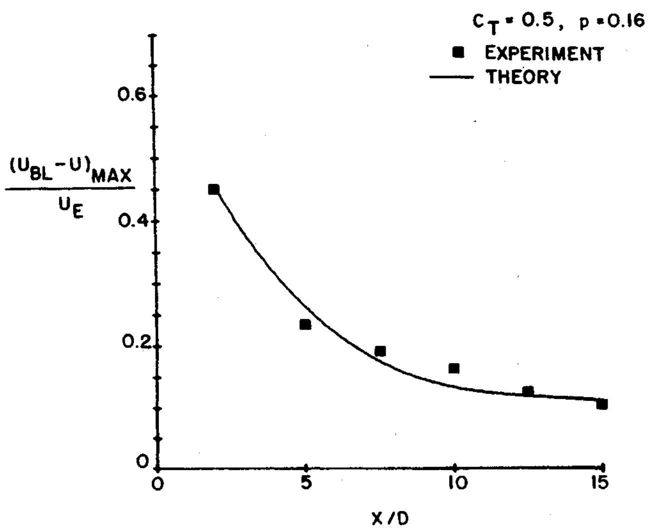

Fig. 6 Maximum velocity defect as a function of streamwise distance for the case of moderate thrust loading with upstream shear.

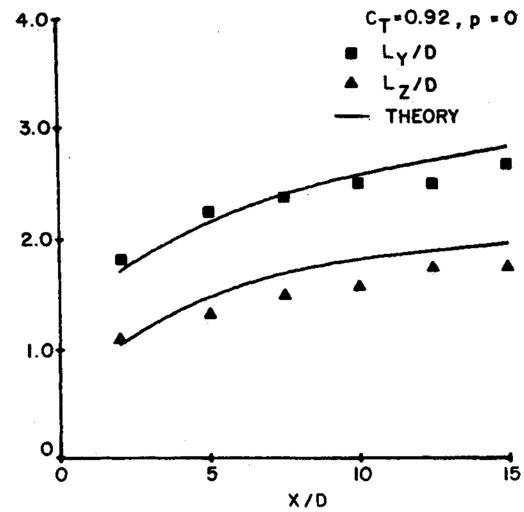

Fig. 8 Wake growth as a function of streamwise distance for the case of high thrust loading and no upstream shear.

typical for free turbulent mixing but extended for three-dimensional cases, as described by Sforza.[33]

The linearized flow equation can then be written in non-dimensional form as

$$u_\xi = \epsilon (u_{\eta\eta} + u_{\zeta\zeta}) \tag{4}$$

where $\epsilon = \epsilon_l / U_E D$. This equation is subject to the initial profile of the wake defect obtained from the experiment and the boundary conditions

$$\lim_{\eta,\zeta \to \infty} u = 0 \quad \text{and} \quad \lim_{\eta \to 0} u = 0$$

which describe the decay of the perturbation far from the wall and at the wall itself.

Numerical Calculations

The Alternating Direction Implicit (ADI) method, developed by Peaceman and Rachford[59] for multidimensional heat flow and diffusion equations was chosen due to its unconditional stability and simplicity. Quantitative comparisons to the experimental results follow in the section devoted to such data. In those comparisons, the velocity defect in the plane of the initial station is used to start the calculation, i.e., the defect velocity distribution in the plane, the maximum velocity defect, and L_Y and L_Z are taken from the experimental results at the first X station. The flowfield and the eddy viscosity are calculated at each station, with a fixed value of the constant $\kappa = 0.063$. The ADI code calculates the entire perturbation flowfield from the measured initial profiles and the fixed value of κ, for all three cases studied here. The value of 0.063 was determined empirically by considering κ as a parameter and selecting the value which gave the best overall fit to the data.

V. Results

Streamwise Defect Decay

Measurements of velocity behind the disk were made in the planes $X/D = 2$, 5, 7.5, 10, 12.5, and 15. The decay of the maximum defect velocity with streamwise distance is shown in Figs. 5-7 for the cases of high thrust (no shear), moderate thrust (with shear), and low thrust (no shear), respectively. Also shown on those figures are the corresponding predictions of the numerical program. The agreement between theory and experiment is quite reasonable, with the theory tending to underestimate the actual velocity defect. It should be noted that the velocity defect $\Delta U_m = (U_{BL} - U)_{\max}$ decays quite rapidly until it reaches values around 0.2 U_E, whereafter the decay is much slower. Indeed, in the very lightly loaded case, $C_T = 0.26$, the initial perturbation itself is very small, and the decay rate is very slow throughout.

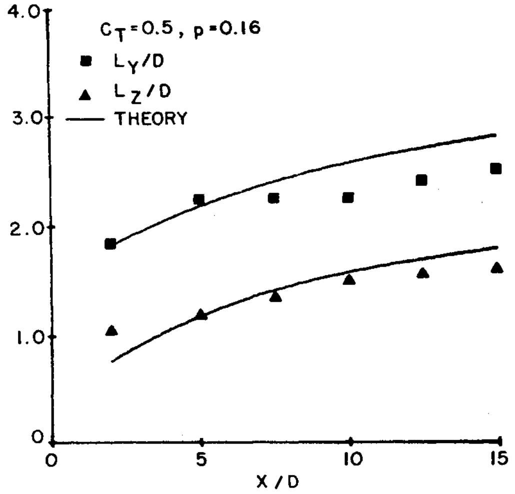

Fig. 9 Wake growth as a function of streamwise distance for the case of moderate thrust loading with upstream shear.

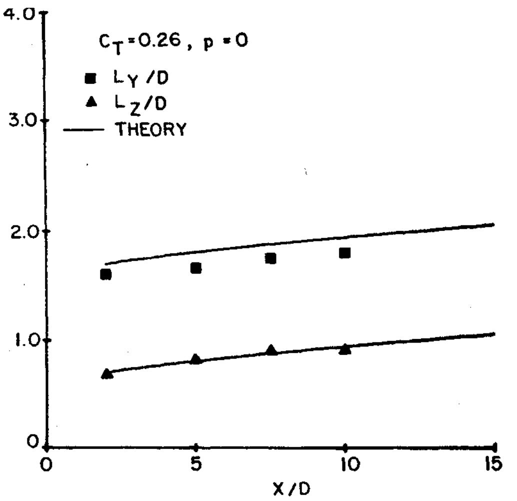

Fig. 10 Wake growth as a function of streamwise distance for the case of low thrust loading with no upstream shear.

In the case of a practical WTG application, one is most interested in the velocity at hub height, because wind turbine characteristics are referenced thereto. If one WTG is located in the wake of another, it is important to know the hub height velocity in that wake. This suggests that we compare the velocity at hub height in the wake, $U_{H,W}$, to that upstream, U_H, in terms of the variables used here. Since the velocity gradients in the wake are not steep and the maximum velocity defect occurs near hub height, $Y/D=1.0$, we may use the approximation

$$U_{H,W}/U_H \simeq 1-(\Delta U_m/U_H) = 1-(\Delta U_m/U_E)/(U_H/U_E)$$

This equation illustrates that both the maximum velocity defect as presented here, and the velocity gradient in the undisturbed flow are important in determining the ratio of hub height velocity in the wake to that in the upstream flow. For example, although the high thrust case shows a larger defect at $X/D=10$ than the moderate thrust case, they both would have about the same value of $U_{H,W}/U_H$, since for the former $U_H/U_E=1.0$ while for the latter it is about 0.8.

Wake Growth

The growth of the wake in the two principal planes is given by L_Y and L_Z. The experimental results obtained for these two quantities are shown in Figs. 8-10 for the cases of high thrust (no shear), moderate thrust (with shear), and low thrust (no shear), respectively. Also shown on these figures are the corresponding predictions of the numerical program. Again, the agreement between theory and experiment is quite reasonable, with the theory tending to overestimate the actual values.

The lines on the figures which describe the theoretical results are actually curves fitted to the results computed at the various downstream stations. In several cases the curves calculated do not pass through the initial data points. This apparent discrepancy is merely due to the poor fit of the computer-plotted curves in the vicinity of the initial points. The initial data used for the computations were those actually measured at the initial station.

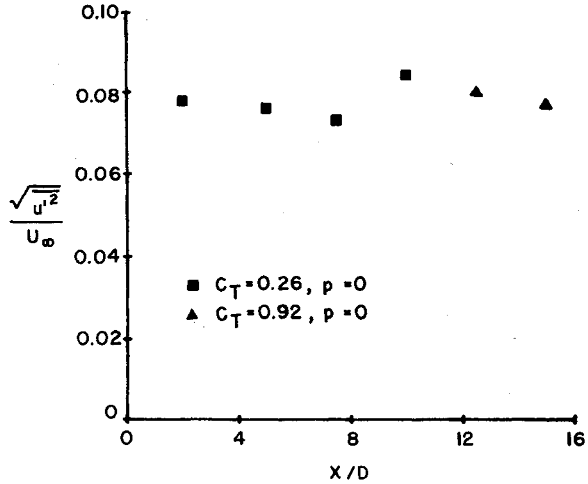

Fig. 11 Relative turbulence intensity in the undisturbed flow as a function of downstream distance for the high and low thrust loading cases with no upstream shear.

Wake Turbulence

An indication of the relative turbulence intensity in the present study is given by the results shown in Fig. 11. There the freestream relative turbulence intensity is shown to be about 8% throughout the field covered by the experiment. Such a value is not untypical of atmospheric boundary layers, although much higher and lower values can be achieved under normal field conditions. We expect that the freestream turbulence will affect the constant κ in the eddy viscosity model, with higher intensities yielding higher values of κ, and lower intensities yielding lower values of κ than the one used in the present study, i.e., $\kappa=0.063$. Actually determining the relationship between κ and intensity is also relegated to future investigations.

VI. Concluding Remarks

We have presented results of an investigation of the wake flow behind simulated WTG units. The study was both experimental and theoretical in nature and included a review of relevant work in the field. We have shown that the wake properties are similar to those expected on the basis of classical turbulent shear flow experience and that relatively simple turbulent flow analyses may be used with some success in describing the flow. In particular, a numerical analysis of the flow based on classical wake flow linearization and a locally calculated eddy viscosity model has been shown to provide a good description of flowfield behavior when compared to the experiments.

Since the experiments covered a fairly broad range of flow conditions, and freestream turbulence levels are comparable to those in the field, it is expected that the approach and the results of the present study can be used to develop methods for adequately predicting full-scale WTG wake flows.

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